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The Gain-Loss Spread: A New and Intuitive Measure of Risk

by Javier Estrada, IESE Business School*

he standard deviation is arguably the most widely used measure of risk.¹ The problem is that, although very widely used, the magnitude is far from intuitive. After all, if the annual returns of an asset have a standard deviation of 20%, what does that number exactly mean? Technically, 20% is the square root of the average quadratic deviation from the arithmetic mean return. But that neither sounds nor is very intuitive.

Because this technically correct interpretation of the standard deviation does not seem to be very insightful, most investors usually resort to thinking about the magnitude in relative terms. Thus, an asset with a volatility of 20% is riskier than one with a volatility of 15%, but less risky than one with a volatility of 25%. Although this approach may be useful when considering the *relative* risk of different assets, the significance of the actual magnitude is still not clearly understood.

This lack of intuitiveness is far from harmless. Daniel Goldstein and Nassim Taleb found that finance practitioners seem to confuse the standard deviation with the mean absolute deviation, which leads them to underestimate risk.² The underestimation ranges between 25% in normally distributed assets and up to 90% for assets with fat tails.

Problems with the standard deviation do not end there. As is well known, when assets display skewness or kurtosis (i.e., "fat tails"), the standard deviation is at best limited and at worst misleading as a measure of risk. Furthermore, most investors associate risk not necessarily with volatility, but more narrowly with bad outcomes, such as losing money, or the probability of losing money or falling short of a target return. In other words, investors tend to associate risk with the downside they face, which may help explain the increasing popularity of measures of downside risk.³ For this reason, the downside should be an *explicit* part of any measure that intends to reflect the way investors think about and assess risk. The goal of this article is to propose a new measure of risk that is both intuitive and based on magnitudes that investors consider relevant when assessing risk. This measure is the *gain-loss spread* (GLS), which takes into account the probability of a loss, the average loss, and the average gain.

As the evidence reported below shows, the GLS is highly correlated with the standard deviation of returns, thus providing basically the same information, yet with more insight than the most widely used measure of risk. Furthermore, the evidence shows that: (1) the GLS is more correlated with mean returns than both the standard deviation and beta, thus providing a tighter link between risk and return; and (2) it is better able to discriminate between high-return and low-return portfolios than beta and equal to or better than the standard deviation, and therefore is a useful tool for portfolio selection.

A (Very) Brief Review of Measures of Risk⁴

Harry Markowitz proposed the first formal definition of risk, the standard deviation of returns.⁵ This magnitude is a measure of total risk and aims to capture dispersion around the mean return. The higher the dispersion, the higher the uncertainty, and therefore the higher the risk of the asset considered.

Part of this total risk can be eliminated through diversification by pooling assets into a portfolio. The risk that cannot be diversified away is the systematic component and is measured by beta.⁶ This magnitude aims to capture volatility relative to the market and measures whether an asset magnifies or mitigates the market's fluctuations.

Markowitz also pioneered the use of downside risk measures by supporting the "semideviation," which measures volatility below a chosen benchmark. Other measures of downside risk include the "lower partial moment," which basically generalizes the semideviation;⁷ Value-at-Risk or

^{*} I would like to thank Manolo Campa, Jennifer Conrad, Roger Koenker, and Rawley Thomas for their comments. Gabriela Giannattasio provided valuable research assistance. The views expressed below and any errors that may remain are entirely my own.

^{1.} According to financial theory, the appropriate measure of risk is the standard deviation when considering an asset in isolation, and beta when considering an asset that is part of a fully diversified portfolio. Practitioners typically use these two magnitudes accordingly.

^{2.} See Goldstein and Taleb (2007).

^{3.} See, for example, Estrada (2002, 2004, 2006, 2007, 2008) and references therein.

^{4.} This section simply highlights some risk magnitudes that have been proposed to assess it. Both Bernstein (1996) and Holton (2004) provide insightful historical

perspectives on the evolution of the concept of risk in finance.

^{5.} See Markowitz (1952, 1959).

^{6.} Variations of CAPM and beta were independently derived by Sharpe (1964), Lintner (1965), and Mossin (1966), with credit also given to Treynor (1961), who never published his work.

^{7.} See Bawa (1975) and Fishburn (1977). The semideviation with respect to a chosen benchmark B ($\Sigma_{\rm B}$) is given by $\Sigma_{\rm g}=\{(1/T)\cdot\Sigma_{\rm t}\,{\rm Min}(R_{\rm t}-B)^2\}^{1/2}$, where R denotes returns, T the number of observations, and t indexes time. The lower partial moment with respect to B (LPM_{\rm B}) is given by LPM_{\rm g}=(1/T)\cdot\Sigma_{\rm t}\,{\rm Min}(R_{\rm t}-B)^{\rm a}), where a is the degree of the lower partial moment. Thus, the lower partial moment does not restrict the deviations with respect to B to be quadratic and in that sense generalizes the semideviation.

Exhibit 1 MSCI World, 1988–2007

and acc	and account for capital gains and dividends. All returns in %.												
Year	Return	Year	Return	Year	Return	Year	Return						
1988	24.0	1993	24.9	1998	22.0	2003	34.6						
1989	17.6	1994	5.0	1999	26.8	2004	15.8						
1990	-16.5	1995	19.5	2000	-13.9	2005	11.4						
1991	19.9	1996	13.2	2001	-15.9	2006	21.5						
1992	-4.2	1997	15.0	2002	-19.0	2007	12.2						

This exhibit shows the annual returns of the MSCI World index between 1988 and 2007. Returns are in dollars

VaR, pioneered at J. P. Morgan, which measures the worst expected outcome over a chosen time horizon at a chosen level of confidence; and "downside beta," which measures whether an asset magnifies or mitigates the market's downside fluctuations.8

Other risk measures relevant for optimal portfolio selection include the mean absolute deviation, the stable dispersion measure, Gini's mean difference, conditional value at risk, and the mini-max.9 It's worth pointing out here that researchers in behavioral finance, argue that risk is a concept too complicated to be summarized by a single number, and therefore recommend using not just one but several factors.¹⁰

The Gain-Loss Spread: An Example

A simple example illustrates the underlying logic behind the gain-loss spread. Exhibit 1 displays the annual returns of the MSCI World index-the most widely used proxy for the world market (equity) portfolio-over the 20-year period 1988 to 2007. The arithmetic mean and standard deviation of these returns are 10.7% and 15.7%, respectively, with the latter figure being the square root of the average quadratic deviations around the mean. By itself, this 15.7% provides little insight into the risk of investing in the world market portfolio.

A good starting point in the development of a more insightful risk measure is to explicitly consider the variables that investors care about when assessing the risk of an asset. Two of these variables focus on the downside. Investors are typically concerned about the probability of suffering losses, a percentage that can be estimated as the proportion of periods in which an asset-generated negative returns. Assuming that the short history of returns in Exhibit 1 is representative of long-term behavior, the probability of an annual loss when investing in the world market portfolio is 25%. This number

simply follows from the fact that this index delivered negative returns in 5 years (1990, 1992, 2000, 2001, and 2002) out of 20.

The second downside variable investors care about is the size of the potential losses, a quantity that can be estimated with the mean return over the periods in which the asset delivered negative returns. In Exhibit 1, the average annual loss when investing in the world market portfolio is -13.9%. This number is the mean return for the five years in which the market went down: (-16.5%, -4.2%, -13.9%, -15.9%, -19.0%)/5 = -13.9%

The probability of a loss and the average loss together lead to the expected loss, which for the world market portfolio is (25%)(-13.9%) = -3.5%. Note that this figure is not the expected annual return, but the expected annual loss. Note also that this figure accounts for both the probability and the size of annual losses.

Similar calculations can assess the upside of an asset. Thus, the short history of returns in Exhibit 1 suggests that, when investing in the world market portfolio, the probability of an annual gain is 75% and the average annual gain is 18.9% (the mean return of the 15 years in which the market went up). Consequently, the expected gain is (75%)(18.9%) = 14.2%.

Finally, the gain-loss spread (GLS) for the world market portfolio is simply the difference between the expected gain and the expected loss: GLS = 14.2% - (-3.5%) = 17.6%. Why is a GLS of 17.6% more insightful than a standard deviation of 15.7%? While there is little intuition behind 15.7%—the square root of an average quadratic deviation-the 17.6% reflects the spread between the upside and the downside, each of which reflects what can be expected in terms of gains and losses from investing in the world market portfolio. In short, the GLS is an insightful metric because it combines three

^{8.} See Estrada (2002, 2007). Hogan and Warren (1974), Bawa and Lindenberg (1977), and Harlow and Rao (1989) all propose CAPM-like models based on different definitions of downside beta. Nawrocki (1999) provides a brief history of downside risk measures and Estrada (2006) provides a primer on the topic aimed at practitioners

^{9.} See Biglova et al. (2004) for a review.

^{10.} See, for example, Fisher and Statman (1999).

Exhibit 2 S&P-500, 1900-2007

This exhibit shows information about the annual and monthly return series of the S&P-500 index over the 1900-2007 period. The first four magnitudes show the number of observations (T), the arithmetic (AM) and geometric (GM) mean return, and the standard deviation (SD). The next three show the probability of a loss (p_L), the average loss (A_L), and the expected loss (E_L). The next three show the probability of a gain (p_G), the average gain (A_G), and the expected gain (E_G). The last magnitude is the gain-loss spread (GLS). All magnitudes as defined in section 3.2. Returns are in dollars and account for capital gains and dividends. All figures but T in %.

	т	AM	GM	SD	P _L	AL	EL	P _G	A _G	E _G	GLS
Annual	108	11.6	9.8	19.6	27.8	-13.5	-3.7	72.2	21.3	15.4	19.1
Monthly	1296	0.9	0.8	5.1	38.3	-3.6	-1.4	61.5	3.7	2.3	3.7

variables that investors consider relevant when assessing the risk of an asset: the probability of a loss, the average loss, and the average gain.

Analytical Framework

The previous example shows that calculating a GLS is not only insightful but also very simple; this section formally introduces this measure of risk. As usual, the formalization assumes that the magnitudes that make up the GLS are estimated from historical data. This, of course, does not prevent a subsequent adjustment of these magnitudes to reflect an investor's views about the future.

Consider an asset with returns R_t , where the subscript t indexes time. Assume that of the T periods for which returns are available, the asset delivers a loss $L_t = R_t < 0$ in N periods and a gain $G_t = R_t > 0$ in M periods, such that N+M = T. The probability of a loss (p_t) is then defined as

$$\mathbf{p}_{\mathrm{I}} = \mathrm{N/T} , \qquad (1)$$

and the probability of a gain as $p_G = M/T = 1-p_L^{.11}$

The average loss (A_L) is defined as the mean return over the N periods in which the asset delivered a negative return:

$$A_{L} = (1/N) \cdot \sum_{t=1}^{N} L_{t}.$$
 (2)

Similarly, the average gain (A_G) is defined as the mean return over the M periods in which the asset delivered a positive return:

$$A_{G} = (1/M) \cdot \sum_{t=1}^{M} G_{t}.$$
 (3)

The expected loss (E_1) builds on (1) and (2) and is defined

as
$$E_{L} = p_{L} \cdot A_{L} = (1/T) \cdot \sum_{t=1}^{N} L_{t}.$$
 (4)

Similarly, the expected gain (E_{c}) is defined as

$$\mathbf{E}_{\mathrm{G}} = \mathbf{p}_{\mathrm{G}} \cdot \mathbf{A}_{\mathrm{G}} = (1/\mathrm{T}) \cdot \sum_{t=1}^{\mathrm{M}} \mathbf{G}_{t}.$$
 (5)

Finally, the gain-loss spread (GLS) is equal to the difference between the expected gain and the expected loss:

$$GLS = E_G - E_L = (1/T) \cdot \sum_{t=1}^{M} G_t - \sum_{t=1}^{N} L_t .$$
 (6)

As this expression clearly shows, calculating a GLS is very simple.¹² And as the example in the previous section hopefully shows, it provides more insight about the risk of an asset than the widely used standard deviation.

Another Example

In order to add further perspective on the gain-loss spread, Exhibit 2 shows the GLS and related magnitudes for the S&P-500 over 1900-2007. The S&P delivered annual losses in 27.8% of the years (30 years), with an average annual loss of -13.5%, resulting in an expected annual loss of -3.7%. The expected annual gain of 15.4% results from an average annual gain of 21.3% in the 72.2% of the years (78 years) in which the S&P delivered positive returns. The annual GLS of 19.1% reflects the spread between the expected gain of 15.4% and the expected loss of -3.7%.

The monthly GLS of 3.7%, in turn, results from the spread between an expected monthly gain of 2.3% and an expected monthly loss of -1.4%. The latter results from the 38.3% of the months in which the S&P delivered losses and

^{11.} It is implicitly assumed that the asset delivers either gains or losses; hence, there are no periods in which $R_t = 0$. This is plausible for most assets even at the daily frequency. Still, it is trivial to accommodate in this framework S periods with $R_t = 0$, such that N+M+S = T.

^{12.} Note that although $\rm E_{g}-\rm E_{L}$ yields the GLS, $\rm E_{g}+\rm E_{L}$ yields the arithmetic mean return.

Exhibit 3 Cross-Section Analysis – Countries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gain-loss spread (GLS). The countries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each country. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticity-consistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.68.

	Constant	t-stat	SD	t-stat	Beta	t-stat	GLS	t-stat	R ²
Panel A	0.003	2.265	0.136	7.746					0.556
	0.008	2.906			0.008	2.607			0.178
	0.002	1.102					0.205	7.962	0.578
Panel B	0.001	0.795	-0.043	-0.426			0.269	1.791	0.579
	0.001	0.452			0.002	0.776	0.194	7.898	0.583
Panel C	0.001	0.297	-0.039	-0.393	0.002	0.761	0.252	1.755	0.584

an average monthly loss of -3.6%; the former results from the 61.5% of the months in which the S&P delivered average gains of 3.7%.¹³

Finally, comparing the annual GLS of the S&P (19.1%) and that of the world market portfolio (17.6%) leads to the conclusion that the U.S. market is riskier than the world market. This is not surprising given that the latter is more diversified, but it is reassuring that the GLS points in the expected direction.

Generalizing Gains and Loses

The discussion in the previous sections defines gains and losses as positive and negative returns, which implicitly sets 0 as the benchmark around which gains and losses are measured. However, some investors may be interested in benchmarks other than 0, such as the rate of inflation, the risk-free rate, or a target return.

The framework presented above can be easily generalized to define gains and losses with respect to any arbitrary benchmark B. In this case, gains and losses can be redefined as $G_t = R_t - B > 0$ and $L_t = R_t - B < 0$, respectively. These definitions may be useful to investors that, for example, have a target return they want to meet on a periodic basis.

This relative definition of gains and losses can be further generalized to account for a time-varying benchmark B_t . In this case, gains and losses can be redefined as $G_t = R_t - B_t > 0$ and $L_t = R_t - B_t < 0$, respectively. These definitions may be useful to investors that, for example, focus on real returns (in which case B_t would be the periodic rate of inflation) or on returns above the periodic risk-free rate.

The Evidence

Statistical Significance: The Cross-Section of Returns

In order to test the ability of the GLS to explain the crosssection of returns, I used the entire MSCI database of countries and industries. The database contains monthly data on 49 countries (22 developed and 27 emerging) and 57 industries. The full sample period available for every country and industry was used in the estimations. Although not all series start at the same time, in all cases the data goes through December 2007. Exhibit A1 in the appendix lists all the countries and industries in the sample, the month in which return data begins for each, and some summary statistics. Exhibit A2, also in the appendix, reports the GLS and related metrics for all the countries and industries in the sample.

The first step of the analysis consisted of estimating, over the whole sample period available for each variable, the (arithmetic) mean return, the standard deviation and beta with respect to the MSCI World index, and the GLS of every country and industry. Subsequently, cross-section regressions were run with mean return as the dependent variable and combinations of the three risk variables (standard deviation, beta, and GLS) as independent variables. These regressions were run across countries, across industries, and across a pooled sample of countries and industries. Exhibit 3 summarizes the results for the cross-section of countries.

Panel A shows the results of simple regressions in which mean returns are regressed on each of the three risk variables, one at a time. As this panel shows, all three variables have the expected sign and are statistically significant. Of the three, the GLS has the highest explanatory power measured by the R². Panel B shows that when the GLS and the standard

^{13.} The S&P delivered neither gains nor losses (R $_{\rm t}$ =0%) in 2 of the 1296 months in the sample.

Exhibit 4 Cross-Section Analysis – Industries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gain-loss spread (GLS). The industries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each industry. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticity-consistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.67.

	Constant	t-stat	SD	t-stat	Beta	t-stat	GLS	t-stat	R ²
Panel A	0.005	4.468	0.065	2.901					0.133
	0.008	4.546			0.001	0.460			0.009
	0.004	3.565					0.108	3.708	0.169
Panel B	0.001	0.307	-0.535	-2.455			0.889	3.001	0.300
	0.004	4.140			-0.006	-3.772	0.231	5.710	0.306
Panel C	0.001	0.804	-0.462	-2.381	-0.005	-3.850	0.888	3.407	0.401

Exhibit 5 Cross-Section Analysis – Countries and Industries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gainloss spread (GLS). The countries and industries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each country and industry. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticityconsistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.66.

Constant	t-stat	SD	t-stat	Beta	t-stat	GLS	t-stat	R ²
0.002	2.508	0.132	8.937					0.510
0.009	4.887			0.003	1.648			0.044
0.001	1.041					0.199	9.465	0.539
-0.000	-0.043	-0.157	-1.350			0.428	2.578	0.550
0.003	2.101			-0.003	-2.536	0.227	9.447	0.570
0.002	1.076	-0.152	-1.373	-0.003	-2.547	0.448	2.758	0.581
	Constant 0.002 0.009 0.001 -0.000 0.003 0.002	Constant t-stat 0.002 2.508 0.009 4.887 0.001 1.041 -0.000 -0.043 0.003 2.101 0.002 1.076	Constant t-stat SD 0.002 2.508 0.132 0.009 4.887 - 0.001 1.041 - -0.000 -0.043 -0.157 0.003 2.101 -	Constant t-stat SD t-stat 0.002 2.508 0.132 8.937 0.009 4.887 - - 0.001 1.041 - - -0.000 -0.043 -0.157 -1.350 0.003 2.101 - -	Constant t-stat SD t-stat Beta 0.002 2.508 0.132 8.937 0.009 4.887 . 0.003 0.001 1.041 . . -0.000 -0.043 -0.157 -1.350 0.003 2.101 . . .0.003 0.002 1.076 -0.152 -1.373 .0.003	Constant t-stat SD t-stat Beta t-stat 0.002 2.508 0.132 8.937 1.648 0.009 4.887 0.003 1.648 0.001 1.041 1.648 -0.000 -0.043 -0.157 -1.350 0.003 2.101 -0.003 -2.536 0.002 1.076 -0.152 -1.373 -0.003 -2.547	Constant t-stat SD t-stat Beta t-stat GLS 0.002 2.508 0.132 8.937	Constant t-stat SD t-stat Beta t-stat GLS t-stat 0.002 2.508 0.132 8.937 1.648 -

deviation are jointly considered, only the GLS is significant and has the expected sign; the same is true when the GLS and beta are jointly considered. Finally, panel C shows that when all three risk variables are jointly considered, only the GLS is significant and has the expected sign. Therefore, Exhibit 3 suggests that when trying to explain the mean returns across countries, the GLS outperforms both the standard deviation and beta.

Exhibit 4 shows the results of a similar analysis for the cross-section of industries. When each risk variable is considered individually (panel A), only the GLS and the standard deviation are significant and have the expected sign, and the GLS has higher explanatory power. When the GLS is jointly considered first with the standard deviation and then with beta (panel B), only the GLS is significant and has the

expected sign. Finally, when all three are considered together (panel C), only the GLS is significant and has the expected sign (the standard deviation and beta have the wrong sign and are significant). Thus, Exhibit 4 suggests that when explaining mean returns across industries, the GLS again outperforms both standard deviation and beta.

Finally, Exhibit 5 shows the results of a pooled analysis of countries and industries, again with similar results. When considered individually (panel A), the GLS is significant, has the expected sign, and achieves the highest explanatory power. When the GLS is jointly considered with either the standard deviation or beta (panel B), only the GLS is significant and has the expected sign. And when all three variables are jointly considered (panel C), only the GLS is significant and has the expected sign.

Exhibit 6 Return Spreads

This exhibit shows mean returns and spreads in mean returns. Countries, industries, and countries and industries combined were ranked by their standard deviation (SD), beta with respect to the MSCI World index (Beta), and gain-loss spread (GLS). In each case, three equally weighted portfolios were formed, the top third with the riskiest assets (P1) and the bottom third with the least risky assets (P3), and their mean returns were calculated. S denotes the spread between the mean return of P1 and P3, and AS denotes the annualized spread. The countries and industries in the sample are those shown in Exhibit A1 in the appendix. All parameters calculated from monthly returns and over the whole sample period available for each country and industry. Returns are in dollars and account for capital gains and dividends. All returns in %.

		Countries				Industries				Countries & Industries			
	P1	P3	S	AS	P1	P3	S	AS	P1	P3	S	AS	
SD	1.9	1.1	0.8	9.8	1.0	0.9	0.1	1.7	1.7	0.9	0.7	8.9	
Beta	1.8	1.5	0.4	4.3	0.9	1.0	_0.1	_1.0	1.4	1.2	0.1	1.4	
GLS	2.0	1.1	0.9	11.4	1.0	0.9	0.2	2.0	1.7	0.9	0.8	9.4	

The results reported in Exhibits 3–5 all point in the same and unequivocal direction: when explaining mean returns across countries, across industries, and across countries and industries combined, the GLS outperforms both the standard deviation and beta. Furthermore, because the GLS is almost perfectly correlated with the standard deviation, it provides basically the same information about risk.¹⁴ In short, the GLS provides a more insightful measure of risk and a tighter link between risk and return than do both the standard deviation and beta.

Economic Significance: Return Spreads

The evidence just discussed clearly supports the GLS as a risk measure, and for this reason an interesting question to ask is whether the GLS is not only statistically significant, but also *economically* significant. In other words, does a portfolio of high-GLS assets outperform a portfolio of low-GLS assets by a substantial margin? Is this margin larger than that between the high- and low-risk portfolios when risk is measured by standard deviation and beta? We turn to these questions next.

As before, the first step of the analysis consisted of estimating, over the whole sample period available for each variable, the (arithmetic) mean return, the standard deviation, beta with respect to the MSCI World index, and the GLS of every country and industry in the sample. Subsequently, all countries were ranked by their standard deviation, and three equally weighted portfolios were formed; the top third with the most volatile countries (P1) and the bottom third with the least volatile countries (P3). Finally, the spread in mean monthly returns (S) between these two portfolios was calculated and subsequently annualized (AS). The same process was repeated for countries ranked by beta and GLS; and then repeated again for industries, and for a pooled sample of countries and industries. All results are summarized in Exhibit 6.

The results for countries show that the spread in mean monthly returns between the most and least volatile portfolios is 0.8%, or 9.8% annualized. That spread is higher than the one between high- and low-beta portfolios (4.3% annualized), but lower than the spread between high- and low-GLS portfolios (11.4% annualized). This last figure shows that differences in GLS may span very large differences in mean returns, and suggests that this risk measure can form the basis of a profitable investment strategy. Furthermore, the 1.6% (11.4%–9.8%) differential spread between the GLS and the standard deviation is far from negligible.

The results for industries, though less spectacular, confirm the ability of the GLS to discriminate between high- and low-return assets. The annualized spread between high-GLS and low-GLS portfolios is 2.0%, higher than that for the standard deviation (1.7%) and beta (surprisingly –1.0%). Finally, the pooled sample of countries and industries strengthens the previous results. The annualized spread between high- and low-GLS portfolios is a substantial 9.4%, higher than those spanned by the standard deviation (8.9%) and beta (1.4%).

In short, these results show that the GLS is better able to discriminate between high- and low-return portfolios than both the standard deviation and beta. Furthermore, the spreads spanned by the GLS are substantial from an economic point of view, ranging from a low of 2% a year for industries to a high of 11.4% a year for countries.

14. The correlation between the GLS and the standard deviation is $0.99\ \text{across}$ countries, across industries, and across countries and industries combined. The

correlation between the GLS and beta, in turn, is 0.47 across countries, 0.78 across industries, and 0.50 across countries and industries combined.

Exhibit 7 Investable Strategies

This exhibit shows the results of two investable strategies. For each country and industry in the sample the standard deviation (SD), beta with respect to the MSCI World index (Beta), and gain-loss spread (GLS) were estimated between the beginning of data coverage and Dec/99. Countries were then ranked by their estimated SDs and allocated into three equally weighted portfolios; subsequently, \$100 was invested in the countries with high SDs (the top third of the ranking), in one case as a long-only position and in the other as a long-short position with the proceeds coming from shorting the countries with low SDs (the bottom third of the ranking); the long-only and long-short portfolios were held through Dec/07, at which point their terminal value (TV), arithmetic mean return (AM), geometric mean return (GM), risk measures (SD, Beta, and GLS), and risk-adjusted return (RAR) were calculated. The same process was repeated after ranking countries by beta and by GLS; and then repeated again after ranking industries by SD, beta, and GLS. RAR is defined as the ratio between AM and the indicated risk measure. "Long-Only" denotes the portfolios with high SD, beta, and GLS, and "Long-Short" denotes portfolios short low-risk assets and long high-risk assets. The countries and industries in the sample are those shown in Exhibit A1 in the appendix. All magnitudes but TV are monthly figures. All figures in % except for TVs (in \$) and betas.

		Long-Only			Long-Short	
	SD	Beta	GLS	SD	Beta	GLS
Countries						
TV	359.70	348.01	353.38	159.32	73.26	157.36
AM	1.51	1.46	1.50	0.56	-0.28	0.56
GM	1.34	1.31	1.32	0.49	-0.32	0.47
SD	5.74	5.44	5.88	3.92	3.07	4.07
Beta	1.14	1.13	1.17	0.21	0.35	0.27
GLS	4.97	4.72	5.07	3.17	2.40	3.28
RAR-SD	0.26	0.27	0.25	0.14	-0.09	0.14
RAR-Beta	0.01	0.01	0.01	0.03	-0.01	0.02
RAR-GLS	0.30	0.31	0.30	0.18	-0.11	0.17
Industries						
TV	205.52	158.75	204.37	116.38	66.31	111.36
AM	0.84	0.60	0.83	0.18	-0.38	0.14
GM	0.75	0.48	0.75	0.16	-0.43	0.11
SD	4.13	4.72	4.17	2.27	3.12	2.37
Beta	0.99	1.17	1.00	0.24	0.56	0.19
GLS	3.41	3.87	3.44	1.59	2.37	1.67
RAR-SD	0.20	0.13	0.20	0.08	-0.12	0.06
RAR-Beta	0.01	0.01	0.01	0.01	-0.01	0.01
RAR-GLS	0.25	0.15	0.24	0.12	-0.16	0.08

Investable Strategies

The in-sample analysis of the previous section, although illuminating, does not evaluate the out-of-sample ability of the GLS, relative to that of standard deviation and beta, to discriminate between high- and low-return portfolios. In order to complement the previous analysis, this section evaluates the usefulness of the GLS as a tool for portfolio selection by testing two out-of-sample (investable) strategies.

The first step of the analysis consisted of estimating risk parameters (SD, beta, and GLS) for every country and industry in the sample between the beginning of data coverage and December 1999. Countries were then ranked by their estimated standard deviations and allocated into three equally weighted portfolios. Subsequently, two strategies were implemented, the first consisting of a long-only investment of \$100 in the portfolio of riskiest countries, and the second consisting of a long-short investment: short \$100 the portfolio of least risky countries and long \$100 the portfolio of riskiest countries. Both the long-only and the long-short portfolios were passively held through the end of 2007, at which point they were liquidated.

The same process was then repeated for countries after being ranked by beta and by GLS; and then again for industries. The performance of all these portfolios is summarized in Exhibit 7. Focusing on the long-only strategies first, the exhibit shows that risky portfolios based on the standard deviation and GLS performed similarly and better than those based on beta. In the case of countries, the performance of all three portfolios was similar in terms of terminal wealth, mean return, risk, and risk-adjusted return. For industries, portfolios based on beta performed substantially worse—in terms of all these metrics—than portfolios based on standard deviation and GLS, both of which performed almost identically.

As for the long-short strategies, the most remarkable result is that low-beta portfolios outperformed high-beta portfolios in the case of both countries and industries. But as expected, high-risk portfolios outperformed low-risk portfolios when risk was measured by standard deviation and GLS. Portfolios based on these two measures of risk performed very similarly in terms of terminal wealth, mean return, risk, and risk-adjusted return in the case of both countries and industries.

These results show that the GLS can be used profitably as the basis of an investable strategy, and that its out-ofsample ability to discriminate between high-risk and low-risk portfolios is better than that of beta and comparable to the standard deviation. They therefore strengthen the results of the previous section and confirm the plausibility of the GLS as a proper measure of risk.

Assessment

Risk can be defined in many ways; some even claim that it is even in the eyes of the beholder. Nonetheless, of the many magnitudes devised to assess it, the standard deviation of returns is the most widely used. The problem is that it lacks a clear intuition. The number itself provides little insight into the risk of an asset, and most academics and practitioners typically use it more in relative terms (the larger the standard deviation, the riskier the asset) than in absolute terms (interpreting a specific value of this magnitude).

Compounding this problem is the fact that investors tend to associate risk less with volatility and more with downside factors, such as losing money or the probability of falling short of a target return. For this reason, the downside should be an explicit part of an intuitive risk measure.

The magnitude proposed in this article, the gain-loss spread, is built by putting together variables that investors associate with risk. By combining the probability of suffering losses, and the magnitude of the potential loses and gains, the GLS provides a more intuitive measure of risk. Furthermore, because it is highly correlated with the standard deviation, it basically provides the same information about risk, but it does so by also providing more insight about the risk of an asset.

Admittedly, the GLS is an ad-hoc measure. But it does provide intuitive insight into the risk of an asset; it is based on magnitudes that investors do consider relevant when assessing risk; and it is supported by the evidence. Goldstein and Taleb expressed hope that "one day, finance will adopt a more natural metric than standard deviation." Perhaps the GLS proposed in this article is a tool that can meet that objective.

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Appendix

Exhibit A1 Summary Statistics

This exhibit shows, for the series of monthly returns, the arithmetic mean (AM), standard deviation (SD), and beta with respect to the MSCI World index (Beta) of all the countries and industries in the sample, all calculated between the beginning (Start) and the end (Dec/2007) of each variable's sample period. All country and industry benchmarks are MSCI indices. Returns are in dollars and account for capital gains and dividends. AM and SD in %.

Country	AM	SD	Beta	Start	Industry	AM	SD	Beta	Start	
Developed					Aerospace & defense	1.3	5.4	0.9	Jan/95	
Australia	1.1	6.8	1.0	Jan/70	Air freight & logistics	0.9	4.9	0.8	Jan/95	
Austria	1.1	5.9	0.5	Jan/70	Airlines	0.6	6.0	1.1	Jan/95	
Belgium	1.3	5.4	0.8	Jan/70	Auto components	0.7	4.9	0.9	Jan/95	
Canada	1.1	5.5	1.0	Jan/70	Automobiles	0.8	5.5	1.1	Jan/95	
Denmark	1.3	5.3	0.7	Jan/70	Beverages	0.9	4.2	0.6	Jan/95	
Finland	1.6	9.1	1.4	Jan/88	Biotechnology	0.9	8.5	0.8	Jan/95	
France	1.2	6.3	1.0	Jan/70	Building products	0.5	4.9	0.9	Jan/95	
Germany	1.1	6.0	0.9	Jan/70	Chemicals	1.0	4.5	0.9	Jan/95	
Hong Kong	1.9	10.5	1.2	Jan/70	Commercial banks	0.9	4.9	1.0	Jan/95	
Ireland	1.0	5.6	1.0	Jan/88	Commercial services & supplies	0.5	4.3	1.0	Jan/95	
Italy	0.9	7.1	0.8	Jan/70	Communications equipment	1.2	9.7	2.0	Jan/95	
lanan	11	6.3	1.0	Jan/70	Computers & peripherals	1.2	8.0	1.6	lan/95	
Netherlands	1.3	5.2	1.0	Jan/70	Construction & engineering	0.8	5.3	0.8	Jan/95	
New Zealand	0.8	6.5	0.8	lan/88	Construction materials	0.9	5.0	1.0	lan/95	
Norway	14	75	1.0	lan/70	Containers & packaging	0.3	54	0.9	lan/95	
Portugal	0.8	63	0.8	lan/88	Distributors	0.0	83	13	lan/95	
Singapore	1 /	83	1 1	Jan/70	Diversified financial services	1.0	5.4	1.5	Jan/95	
Snain	1.4	63	0.9	Jan/70	Diversified telecommunication services	0.7	5.4	1 1	lan/95	
Sweden	1.1	6.7	1.0	Jan/70	Electric utilities	1.0	3.0	0.4	Jan/95	
Switzerland	1.4	5.2	0.9	Jan/70	Electronic equipment & instruments	0.5	76	1.5	Jan/95	
	1.1	5.2	1.1	Jan/70	Electronic equipment manufacturers	1.0	5.5	1.5	Jan/95	
	0.0	13	0.0	Jan/70	Electronic equipment & services	1.0	9.5	1.2	Jan/95	
Emorging	0.9	4.5	0.9	Jan/70	Energy equipment & services	0.0	3.0	0.4	Jan/95	
Argonting	27	16.2	07	lan/88	Food/staplos retailing	0.9	3.4	0.4	Jan/95	
Prozil	2.7	10.2	1.6	Jan/00		1.0	3.4 2.0	0.5	Jan/95	
Chile	3.1	15.0	1.0	Jaii/00	Uasith agra aguisment & support	1.0	3.0	0.0	Jan/05	
China	1.0	10.0	0.7	Jan/02	Health care providers & convices	1.0	4.0	0.0	Jan/95	
Colombio	1.0	10.9	1.1	Jan/93	Health care providers & services	0.9	0.0	0.5	Jan/95	
Coloribia Creek Den	1.0	9.5	0.5	Jan/95		0.6	4.0	0.9	Jan/95	
Czech Rep.	1.9	0.0	0.7	Jan/95	Household areducts	0.5	0.1	1.2	Jan/95	
Egypt	2.4	9.2	0.5	Jan/95	Housenola products	1.2	4.6	0.3	Jan/95	
Hungary	2.2	10.0	1.3	Jan/95	Industrial conglomerates	1.1	5.1	1.1	Jan/95	
India	1.5	8.2	0.7	Jan/93	Information technology services	0.3	7.8	1.3	Jan/95	
Indonesia	2.0	15.0	0.9	Jan/88	Insurance	0.9	5.0	1.0	Jan/95	
Israel	1.0	7.2	1.0	Jan/93	Internet catalogue & retail	1.1	9.3	1.2	Jan/95	
Jordan	0.8	5.1	0.1	Jan/88	Internet software services	2.0	17.2	2.3	Jan/95	
Korea	1.3	11.1	1.2	Jan/88	Leisure equipment & products	0.4	4.3	0.6	Jan/95	
Malaysia	1.2	8.8	0.9	Jan/88	Machinery	0.8	5.1	1.0	Jan/95	
Mexico	2.3	9.2	1.1	Jan/88	Marine	1.0	6.3	1.0	Jan/95	
Morocco	1.5	5.3	0.2	Jan/95	Media	0.6	5.5	1.2	Jan/95	
Pakistan	1.4	11.0	0.4	Jan/93	Metals & mining	1.3	6.6	1.1	Jan/95	
Peru	2.1	8.9	0.8	Jan/93	Multi utilities	0.7	6.1	1.0	Jan/95	
Philippines	1.1	9.4	0.9	Jan/88	Multiline retailers	1.0	5.2	0.9	Jan/95	
Poland	2.6	14.7	1.6	Jan/93	Office Electronics	0.7	6.4	1.0	Jan/95	
Russia	3.4	17.1	2.0	Jan/95	Oil, gas & consumable fuels	1.4	4.9	0.7	Jan/95	
South Africa	1.5	7.7	1.1	Jan/93	Paper & forestry products	0.5	5.8	1.0	Jan/95	
Sri Lanka	1.0	10.0	0.3	Jan/93	Personal products	1.3	5.2	0.8	Jan/95	
Taiwan	1.2	10.9	0.9	Jan/88	Pharmaceuticals	1.0	3.9	0.5	Jan/95	
Thailand	1.3	11.4	1.3	Jan/88	Road & rail	0.6	3.8	0.5	Jan/95	
Turkey	2.6	17.3	1.3	Jan/88	Software	1.7	8.8	1.6	Jan/95	
Venezuela	1.7	14.5	0.9	Jan/93	Specialty retail	0.8	5.9	1.1	Jan/95	
					Textiles, apparel & luxury goods	0.9	5.3	1.1	Jan/95	
					Tobacco	1.7	6.6	0.4	Jan/95	
					Trading companies & distributors	0.7	6.7	0.9	Jan/95	
					Transportation infrastructure	1.1	4.8	0.5	Jan/95	
					Water utilities	1.6	5.0	0.3	Jan/95	
					Wireless telecommunication services	1.5	6.7	1.1	Jan/95	

Exhibit A2 GLS and Related Magnitudes

This exhibit shows, for the series of monthly returns summarized in Exhibit A1, the probability of a loss (p_L) , the average loss (A_L) , the average gain (A_G) , and the gain-loss spread (GLS) for all the countries and industries in the sample, all calculated between the beginning and the end of each variable's sample period and for a benchmark of 0%. Returns are in dollars and account for capital gains and dividends. All numbers in %.

Country	p _L	AL	A _G	GLS	Industry	p _L	AL	A_{g}	GLS	
Developed					Aerospace & defense	36.5	-4.1	4.4	4.3	
Australia	43.4	-4.6	5.4	5.1	Air freight & logistics	41.7	-3.3	4.0	3.7	
Austria	42.5	-3.6	4.7	4.2	Airlines	43.6	-4.4	4.5	4.5	
Belgium	37.7	-3.8	4.3	4.1	Auto components	42.3	-3.9	4.0	3.9	
Canada	39.7	-4.0	4.4	4.2	Automobiles	36.5	-4.5	4.0	4.2	
Denmark	39.3	-3.7	4.5	4.2	Beverages	36.5	-3.2	3.3	3.2	
Finland	44.2	-6.1	7.6	7.0	Biotechnology	48.7	-5.3	6.9	6.1	
France	41.4	-4.5	5.2	4.9	Building products	43.6	-3.7	3.8	3.8	
Germany	41.0	-4.3	4.9	4.7	Chemicals	41.0	-3.2	3.9	3.6	
Hong Kong	40.6	-6.6	7.7	7.2	Commercial banks	39.7	-3.6	3.9	3.8	
Ireland	41.3	-4.2	4.6	4.4	Commercial services & supplies	38.5	-3.8	3.1	3.4	
Italy	45.2	-5.1	5.8	5.5	Communications equipment	42.9	-7.3	7.6	7.4	
Japan	44.1	-4.4	5.4	4.9	Computers & peripherals	44.2	-5.8	6.9	6.4	
Netherlands	36.2	-3.8	4.1	4.0	Construction & engineering	44.2	-4.0	4.7	4.3	
New Zealand	44.6	-4.8	5.3	5.1	Construction materials	37.8	-4.1	3.9	4.0	
Norway	43.0	-5.3	6.4	5.9	Containers & packaging	44.9	-4.1	3.9	4.0	
Portugal	43.8	-4.6	5.0	4.8	Distributors	42.9	-6.8	5.2	5.9	
Singapore	40.1	-5.5	5.9	5.8	Diversified financial services	37.2	-4.3	4.2	4.2	
Spain	42.8	-4.4	5.2	4.9	Diversified telecommunication services	39.1	-4.4	4.0	4.2	
Sweden	41.9	-4.7	5.8	5.4	Electric utilities	35.9	-2.4	2.9	2.7	
Switzerland	40.8	-3.5	4.4	4.0	Electronic equipment & instruments	47.4	-5.4	5.8	5.6	
UK	40.1	-4.3	4.8	4.6	Electronic equipment manufacturers	39.1	-4.3	4.4	4.4	
USA	39.3	-3.1	3.6	3.4	Energy equipment & services	44.2	-5.7	7.4	6.7	
Emerging					Food products	34.0	-2.7	2.7	2.7	
Argentina	44.2	-9.0	12.0	10.7	Food/staples retailing	36.5	-2.7	2.7	2.7	
Brazil	39.6	-10.0	11.7	11.0	Gas utilities	37.8	-2.7	3.3	3.1	
Chile	40.8	-4.6	6.3	5.6	Health care equipment & support	35.3	-3.2	3.3	3.3	
China	47.2	-7.8	8.2	8.0	Health care providers & services	38.5	-4.4	4.2	4.3	
Colombia	41.7	-6.8	7.9	7.4	Hotels, restaurants & leisure	39.7	-3.4	3.7	3.6	
Czech Rep.	37.2	-5.9	6.5	6.3	Household durables	44.2	-4.8	4.6	4.7	
Egypt	41.0	-5.4	7.9	6.9	Household products	35.3	-3.3	3.6	3.5	
Hungary	40.4	-6.7	8.3	7.7	Industrial conglomerates	39.1	-3.7	4.1	4.0	
India	43.3	-6.1	7.4	6.9	Information technology services	42.3	-6.2	5.0	5.5	
Indonesia	43.3	-9.0	10.5	9.9	Insurance	40.4	-3.5	3.9	3.7	
Israel	37.2	-6.1	5.2	5.5	Internet catalogue & retail	49.4	-5.8	7.8	6.8	
Jordan	49.2	-3.1	4.5	3.8	Internet software services	42.3	-11.6	12.2	11.8	
Korea	50.0	-6.6	9.3	8.0	Leisure equipment & products	46.2	-3.3	3.6	3.4	
Malaysia	42.9	-5.9	6.4	6.2	Machinery	41.7	-4.0	4.3	4.2	
Mexico	37.9	-6.8	7.9	7.5	Marine	42.9	-4.4	5.1	4.8	
Morocco	39.7	-3.4	4.7	4.2	Media	37.2	-4.5	3.6	4.0	
Pakistan	50.6	-6.6	9.7	8.1	Metals & mining	41.0	-4.9	5.5	5.3	
Peru	39.4	-6.1	7.4	6.9	Multi utilities	34.6	-5.5	3.9	4.5	
Philippines	45.4	-6.6	7.5	7.1	Multiline retailers	38.5	-4.2	4.2	4.2	
Poland	40.6	-8.8	10.3	9.7	Office Electronics	40.4	-5.4	4.9	5.1	
Russia	41.0	-11.4	13.6	12.7	Oil, gas & consumable fuels	34.6	-3.6	4.1	3.9	
South Africa	38.3	-6.0	6.1	6.1	Paper & forestry products	47.4	-4.2	4.6	4.4	
Sri Lanka	46.1	-6.7	7.5	7.1	Personal products	32.1	-4.2	3.9	4.0	
Taiwan	47.9	-7.3	9.0	8.2	Pharmaceuticals	37.8	-3.0	3.4	3.3	
Thailand	44.2	-7.9	8.6	8.3	Road & rail	42.3	-3.0	3.2	3.1	
Turkey	47.1	-11.2	14.8	13.1	Software	39.7	-6.2	6.9	6.6	
Venezuela	51.1	-8.2	12.1	10,1	Specialty retail	38.5	-5.0	4.4	4.6	
Average	42.5	-5.9	7.1	6.6	Textiles, apparel & luxury goods	42.3	-3.8	4.3	4.1	
		515			Tobacco	31.4	-5.8	5.1	5.3	
					Trading companies & distributors	45.5	-5.1	5.6	5.4	
					Transportation infrastructure	35.3	_4.1	3.9	4.0	
					Water utilities	37.2	-3.3	4.5	4.1	
					Wireless telecommunication services	35.9	-5.1	5.3	5.2	
						40 1	_4.4	4.6	4 5	
						-0.1		7.0	7.5	

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